MULTIVARIABLE PLANT SET ESZ'JMATION USING MULTISINUSOIDAL INPUT DESIGNS

David S. Bayard and Fred Y. Hadaegh

Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Drive
Pasadena, CA 91109

ABSTRACT

A frequency domain method is developed for statistical multivariable plant set estimation. The estimation of a plant "set" rather than a point estimate is required to support many methods of modern robust control design. The approach here is based on using multisinusoidal input designs, and acquiring multivariable data from a sequence of SIMO experiments. Results for the multivariable case extend earlier results developed for the S1S0 case. The data is preprocessed using DFT and signal processing methods, and certain key statistical properties of the estimators are presented in the multivariable case. These properties lead to a precise characterization of the plant set to a specified statistical confidence, e.g., $(1 - \alpha) \cdot 100\%$. The significance of this result is that if a robust controller is designed to provide some specified level of stability or performance *for* all *plants lying in the additive uncertainty set*, then with probability $1 - \alpha$ the controller will work as planned *when applied to the true system.*

An advantage of statistical uncertainty characterizations is that they are potentially less conservative than deterministic uncertainty characterizations. For example, the notion that noise disturbances tend to "average out" over time is missing from deterministic bounded noise treatments.

1. INTRODUCTION

Consider the multivariable system with output noise, given by,

$$y(k) = \mathcal{P}(z^{-1})u(k) + \mathbf{v(k)} \tag{1}$$

where $\mathcal{P}(z^{-1})$ is the q-input, r-output multivariable LTI plant and $v \in R^r$ is an output vector disturbance. It is desired to identify this system in the following form,

$$\mathcal{P}(z^{-1}) = P^o(z^{-1}) + \text{AA} \tag{2}$$

where $P^o(z^{-1})$ is a nominal estimate of the true plant $\mathcal{P}(z^{-1})$, and AA is the additive uncertainty defined as $\text{AA} = \mathcal{P} - P^o$. Since the true plant is not known, it is desired to represent the additive uncertainty in the form

$$\text{AA} = \text{AW}_{\text{A}} \tag{3}$$

1

such that A is norm bounded (i. e., such that $||\Delta||_\infty \le 1$) and such that W_A is a minimum-phase transfer function weighting matrix. The filter matrix W_A is then typically incorporated into the control design, to ensure robustness properties over the additive uncertainty Set.

The goal of this paper is to identify a nominal plant estimate P^o, and a weighting filter W_A *such* that the relation $\mathcal{P} = \text{PO} + \Delta W_A$ holds (for some $||\Delta||_\infty \le 1$) to a specified statistical confidence $1 - \alpha$ specified by the designer. It is then a separate problem in modern robust control synthesis to find a compensator C that has desirable stability and performance properties for all plants in the uncertainty set defined by P^o and W_A [1][7].

The rationale is that if C can ensure some level of performance for all plants in the additive uncertainty set defined by *W_A,* then the controller *will work as* designed *when* implemented on *the real plant* with probability $1 - \alpha$. This approach effects a marriage between the hard uncertainty bounds used in modern H_∞ robust control designs, and the soft bounds obtainable using statistical methods.

2. A-PRIORI INFORMATION

The estimation of a plant set requires the specification of certain a-priori information. The assumptions are given explicitly in this section.

First, the following definition will be needed.

Definition 1 A MIMO transfer function $G(z^{-1})$ is said to be in $\mathcal{D}(M,\rho)$ if the impulse response matrix sequence $\{g(kT)\}_{k=0}^\infty$ defined by the Z-transform relation $\sum_{k=0}^\infty g(kT)z^{-k} = G(z^{-1})$ satisfies,

$$\overline{\sigma}(g(kT)) \le M\rho^k$$

for some $\infty > M > 0$ and $1 > \rho \ge 0$. ∎

The main usefulness of Definition **1** is due to the next lemma.

Lemma 1 Let $G(z^{-1}) \in \mathcal{D}(M,\rho)$. Then the derivative of G on the unit circle can be uniformly bounded from above as follows,

$$\overline{\sigma}\left(\frac{dG(e^{-j\omega T})}{d\omega}\right) \le \frac{TM\rho}{(1-\rho)^2}$$

Proof: sce [6]. ∎

The bound in Lemma 1 insures a certain smoothness in G and allows one to overbound errors incurred interpolating frequency data in-bet ween grid points.

Assumption 1. The true plant $\mathcal{P}(z^{-1})$ is a stable unknown linear time-invariant (LTI) g-input r-output multivariable transfer function assumed to be in $\mathcal{D}(M, \text{p})$, where M and ρ are assumed known. ∎

The experiment design is now briefly described. Consider the periodic input excitation design into the n_1th actuator, composed of a harmonically related sum of sinusoids,

$$u_s(k, n_1) = \sum_{i=1}^{n_s} \sqrt{2\alpha_i(n_1)} \cos(\omega_i kT + \phi_i(n_1)) \tag{4}$$

where $\omega_i = 2\pi i/T_p$, $T_p = N_sT, n_s \leq N_s/2$. The power is normalized as,

$$\sum_{i=1}^{n_s} \alpha_i(n_1) = 1 \tag{5}$$

where the relative power in each component { $\alpha_i(n_1) > 0$, $i = 1, ...n_s$ } is assumed specified. In order to minimize peaking in time domain the sinusoids are phased according to Schroeder [20] as,

$$\phi_i(n_1) = 2\pi \sum_{j=1}^{i} j\alpha_j(n_1) \tag{6}$$

(Here, a slightly modified form of the Schroeder phase is used in (6), as derived in Young and Patton [22]). More recent expressions which use the Schroeder design as a starting point for further reducing the crest factor of the multisinusoid signal (4) can also be used [11]. The Schroeder phasing (6) is used here mainly for implementation convenience. The actual choice of phase does not effect the analysis or change any of the main results herein.

Assumption 2 Data for the multivariable case is assumed to be taken by performing q separate single-input multiple-output (SIMO) experiments, using a multisinusoidal excitation of the form (4) with the full number of sinusoids $n_s = N_s/2$ for each experiment. ∎

Assumption 3 The output disturbance $v(k) \in R^r$ can be represented by $v(k) = W(z^{-1})d(k)$ where $d(k) \in R^r$ is a white zero-mean Gaussian vector noise sequence normalized such that $E[d(j)d^T(k)] = \delta_{jk} . I$; $W(z^{-1})$ is a *diagonal matrix* of filters

$$W(z^{-1}) = Diag\left\{ W(z^{-1}, 1), ..., W(z^{-1}, r) \right\} \tag{7}$$

where $W(z^{-1}, n_2)$ is a minimum phase (stable and stably invertible) transfer function, $n_2 = 1,...$)r. ∎

Assumption 4 Data from each SIMO experiment is taken while the system is in periodic steady-state, ∎

Assumption 5: The input period N_sT of the multisinusoidal design (4) is long compared to the time constants of noise filter W and its inverse $W-1$.

In this formulation, the designer has the freedom to choose the frequency shaping $\{\alpha_i(n_1)\}$ and the number of periods of data collected $m(n_1)$ in each SIMO experiment.

3. PREVIOUS RESULTS

Various methods of plant set estimation have been given in the literature. The various approaches can be roughly divided depending upon whether they use time-domain estimation as in Kosut [14], Younce and Rhors [21], Goodwin and Salgado [12], or frequency domain estimation as in Lamaire et. al. [16], Parker and Bitmead [17], Bayard [3], or De Vries and Van den Hof [8][9].

Methods within each category generally differ based on the types of inputs allowed (e.g., sccxmd-order stationary, white noise, periodic, multisinusoidal, persistent exciting, etc.), the types of quantities being estimated (e.g., plant dynamics, noise PSD's, unmodelled dynamics), model parametrization (e. g., pole-zero models, FIR models, etc.), assumptions on the noise (e.g., bounded noise, bounded noise DFT, Gaussian noise, etc.), and the type of a-priori information required (i.e., smoothness priors, open-loop damping, model order, relative degree, etc.).

Related approaches which give hard bounds on the identified model error can also be found in Helmicki, Jacobson and Nett [13] and Gu and Khargonekar [10]. However, these methods are not directly comparable since they start by assuming frequency data is available in a specific form (i .e., with hard error bounds) and do not explicitly separate the error into noise and unmodelled dynamics.

A specific method for S1S0 plant set estimation using multisinusoids has been given in [3] for the case of

(i) Gaussian noise $d(k)$

(ii) Data taken in steady-state

(iii) Noise shaping filter of form $W = \sigma \overline{W}(z^{-1}, 1)$ where filter $W(z^{-1}, 1)$ is known, and σ is a scale factor which can be known or unknown.

These results were generalized by De Vries and Van den Hof [9] as follows,

(i) non-Gaussian noise

(ii) Data not necessarily taken in steady-state

(iii) The weighting filter $W(z^{-1})$ is completely unknown

Eliminating the need for a-priori knowledge of the noise coloring filter $W(z^{-1})$ is particularly useful in practice. The main cost of these improvements is that the expressions obtained in [9] are only valid *asymptotically* (valid as the input period $N_s T$ becomes large). However, exact expressions are difficult to obtain in the case where noise is estimated along with unmodelled dynamics, and asymptotic results of this type may be unavoidable for bridging the gap between theory and practice.

The present paper will extend the S1S0 results in [3] to the multivariable case. In this extension, a philosophy will be used similar to [9], in that a-priori knowledge of W will be avoided at the expense of obtaining only asymptotic results (i.e., valid as $N_s T$ becomes large).

4. MIMO PLANT SET ESTIMATION

An additive error $\Delta_A(z^{-1})$ is used to characterize the mismatch between the true plant $\mathcal{P}(z^{-1})$ and a nominal plant estimate $P^o(z^{-1})$, i.e.,

$$\Delta_A(z^{-1}) = \text{`P}(z^{-1}) - P^o(z^{-1})$$

(8)

It will be useful to define an *additive uncertainty set* as the set of plants $\Omega_A(P^o, \ell_A(\omega))$ associated with a specified overbound $\ell_A(\omega)$ on the additive error, i.e.,

$$\Omega_A(P^o, \ell_A(\omega)) = \{P : \ \overline{\sigma}(P - P^o) \le \ell_A(\omega), \ for \ all \ \omega \in [0, \pi/T]\} \tag{9}$$

This notion is extended to the specification of a *statistical* overbound $\ell_A^{1-\alpha}(\omega)$ in the following definition,

Definition 2 $\ell_A^{1-\alpha}(\omega)$ **is** said to be an *overbound on the additive uncertainty with* statistical confidence $(1 - \alpha) \times 1.00\%$ if,

$$Prob\{\mathcal{P} \in \Omega_A(P^o, \ell_A^{1-\alpha}(\omega))\} \ge 1 - \alpha \tag{10}$$

∎

The significance of this definition is that $\ell_A^{1-\alpha}$ characterizes (to statistical confidence $(1 - \alpha) \times 100\%$) a set in which the true plant $\mathcal{P}$ belongs. Hence, if a robust controller is designed to provide some specified level of performance *for all plants lying in the additive uncertainty set* $\Omega_A(P^o, \ell_A^{1-\alpha}(\omega))$, then with probability $1 - \alpha$ the *controller* will work as planned when *applied to the true system.*

The calculation of a statistical overbound $\ell_A^{1-\alpha}$ from pointwise overbounds is given in the next result.

Lemma 2

Given discrete-time plant $7(-z^{-1}) \in \mathcal{D}(M, \rho)$, assume that noisy frequency domain data $\{\widehat{P}(\omega_i)\}_{i=1}^{N}$ are available on a uniform grid on the unit circle $\omega_i = i\Delta_g$, $i = 1, ..., N$ with grid spacing $\Delta_g = \omega_{i+1} - \omega_i = \frac{\pi}{TN}$. Assume that the accuracy of each data point can be characterized by the quantity ϵ_i such that the event E_i,

$$E_i : \quad \overline{\sigma}(\mathcal{P}(\omega_i) - \widehat{P}(\omega_i)) \le \epsilon_i \tag{11}$$

is satisfied with at least probability $1 - \kappa$ at each grid point i. Here, the events E_i $i = 1, ..., N$ may or may not be jointly statistically independent. Let $S(P, \widehat{\omega})$ be a linear spline interpolant to the data $\{\widehat{P}(\omega_i)\}_{i=1}^{N}$, i.e.)

$$S(\widehat{P}, \omega) = \begin{cases} \widehat{P}(\omega_i) + \frac{(\omega - \omega_i)}{\Delta_g}(\widehat{P}(\omega_{i+1}) - \widehat{P}(\omega_i)) & \text{for } w \in (\omega_i, \omega_{i+1}] \\ \widehat{P}(\omega_1) & \text{for } w \in [0, \omega_1]. \end{cases} \tag{12}$$

and let $P^o(z^{-1})$ be any stable parametric model fit to the data. If $\ell_A^{1-\alpha}(\omega)$ is defined as,

$$\ell_A^{1-\alpha}(\omega) = B_1(\omega) + B_2(\omega) \tag{13}$$

where,

$$B_1(\omega) = \begin{cases} \epsilon_{i+1} + \overline{\sigma}(S(\widehat{P}, \omega) - \widehat{P}(\omega_{i+1})) + \frac{\Delta_g TM\rho}{(1-\rho)^2} & \text{for } \omega \in (\omega_i, \omega_{i+1}] \\ \epsilon_1 + \frac{\Delta_g TM\rho}{(1-\rho)^2} & \text{for } \omega \in [0, \omega_1] \end{cases} \tag{14}$$

$$B_2(\omega) = \overline{\sigma}(\,S(\widehat{P}, \omega) - P^o(\omega)) \tag{15}$$

then, $\ell_A^{1-\alpha}$ is an overbound on the additive uncertainty with statistical confidence,

$$1 - \alpha = \begin{cases} (1 - \kappa)^N & \text{if } E_i, \text{i} = 1, \ldots, \text{N} \text{ are independent} \\ 1 - \kappa N & \text{otherwise} \end{cases} \tag{16}$$

i.e., $\ell_A^{1-\alpha}(\omega)$ satisfies,

$$Prob\{\mathcal{P} \in \Omega_A(P^o, \ell_A^{1-\alpha}(\omega))\} \geq 1 - \alpha$$

Proof: The result proved **identically** to Theorem 2.1 in Bayard [3], by replacing absolute values $|.|$ of scalar quantities with maximum singular values $\overline{\sigma}(\cdot)$ of matrix quantities. ∎

Intuitively, the overbound $\ell_A^{1-\alpha}(\omega)$ in (13) can be thought of as the sum of three terms: a *curve fit* error $B_2(\omega)$; an *estimation error at* the grid points ϵ_i; and an *interpolation error* between grid points $B_1(\omega) - \epsilon_i$.

Values for M and ρ will be assumed known a-priori (they may be known from the physics of the process, practical experience, or can be found by impulse or step response experiments). Systematic methods for finding multivariable data $\{\widehat{P}(\omega_i), \epsilon_i\}_{i=1}^N$ with *the* desired properties in Lemma 2 will be the main focus of the remainder of this paper.

5. DATA ACQUISITION AND PROCESSING

Consider a single-channel input design $u_s(k, n_1)$ applied to the n_1 th actuator, with design weights $\alpha_i(n_1)$, ?2] = 1, ..., q of the multisinusoidal form (4). The system is allowed to reach periodic steady-state, at which time the plant response at the n_2th sensor is denoted as $y_s(k, n_2, n_1)$ for each sensor $n_2 = 1, \ldots, r$. Consider breaking the data into windows which are precisely one period long in length, and denote the output data from the ℓth period as,

$$y_s^\ell(k, n_2, n_1) = y_s(k + (\ell - 1)N_s, n_2, n_1) \tag{17}$$

for $k = 0, \ldots, N_s - 1$ and $\ell = 1, \ldots, m(n_1)$. It is assumed that $m(n_1)$ periods of output data (i.e., $y_s^\ell(k, n_2, n_1), \ell = 1, \ldots, m(n_1)$), are collected at steady-state.

This process is repeated for q SIMO experiments $n_1 = 1, \ldots, q$, corresponding to a separate experiment from each of the system's q actuators. The frequency shaping $\{\alpha_i(n_1)\}$ of the multisinusoidal design (4) and number of periods $m(n_1)$ of data collected, can be chosen differently for each SIMO experiment. For notational convenience, the total number of data windows acquired over *all* experiments is defined by,

$$\overline{m} \triangleq \sum_{n_1=1}^q m(n_1) \tag{18}$$

Under Assumptions 1-5 one can construct the following estimates and statistical distributions [5],

DFT Plant Estimator

$$\widehat{P}(\omega_i, n_2, n_1) = \frac{\overline{Y}(\omega_i,\ n_2, n_1)}{U(\omega_i, n_1)} \cdot \quad n_1 = 1, ..., q; \quad n_2 = 1, ..., r \tag{19}$$

$$\overline{Y}(\omega_i, n_2, n_1) = \begin{cases} \frac{1}{m(n_1)} \sum_{\ell=1}^{m(n_1)} Y^\ell(\omega_i, n_2, n_1) & \text{for } \omega_i \ \text{'r}\ \omega_{N_s-i} \in \text{'s} \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

$$Y^\ell(\omega_i, n_2, n_1) = \sum_{k=0}^{N_s-1} y_s^\ell(k, n_2, n_1) e^{-j\omega_i kT} \tag{21 CL}$$

$$U(\omega_i, n_1) = \sum_{k=0}^{N_s-1} u_s(k, n_1) e^{-j\omega_i kT} \tag{21.b}$$

It is noted that $\widehat{P}(\omega_i, n_2, n_1)$ is defined only at points of nonzero energy in the input (i.e., for i such that ω_i or ω_{N_s-i} is in u_s).

SIMO-Data Estimates

$$|\widehat{W}(\omega_i, n_2 | n_1)|^2 = \frac{\sum_{\ell=1}^{m(n_1)} |Y(\omega_i, n_2, n_1)\ \text{-,}\ \overline{Y}(\omega_i, n_2, n_1)|^2}{N_s(m(n_1)\ -\ 1)} \tag{22}$$

$$\frac{(2m(n_1)\ \text{-}\ 2)|\widehat{W}(\omega_i, n_2 | n_1)|^2}{|W(\omega_i, n_2)|^2} \sim \chi^2(2m(n_1)\ \text{-}\ 2) \tag{23}$$

$$\frac{\alpha_i(n_1) m(n_1) N_s}{2|\widehat{W}(\omega_i, n_2 | n_1)|^2} \cdot |\widehat{P}(\omega_i, n_2, n_1)\ \text{-}\ \mathcal{P}(\omega_i, n_2, n_1)|^2 \sim F(2, 2m(n_1)\ \text{-}\ 2) \tag{24}$$

MIMO-Data Estimates

$$|\widehat{W}(\omega_i, n_2)|^2 = \frac{\sum_{n_1=1}^{q} \sum_{\ell=1}^{m(n_1)} |Y(\omega_i, n_2, n_1)\ \text{-}\ \overline{Y}(\omega_i, n_2, n_1)|^2}{N_s(\overline{m}\ \text{-}\ q)} \tag{25}$$

$$\frac{(2\overline{m}\ \text{--}\ 2q)|\widehat{W}(\omega_i, n_2)|^2}{|W(\omega_i, n_2)|^2} \sim \chi^2(2\overline{m}\ \text{-}\ 2q) \tag{26}$$

$$\frac{\alpha_i(n_1) m(n_1) N_s}{2|\widehat{W}(\omega_i, n_2)|^2} \cdot |\widehat{P}(\omega_i, n_2, n_1)\ \text{-}\ \mathcal{P}(\omega_i, n_2, n_1)|^2 \sim F(2, 2\overline{m}\ \text{-}\ 2q) \tag{27}$$

Here, $X^2(v)$ denotes a Chi-Squared distribution with ν degrees of freedom and $F(\nu_1, \nu_2)$ denotes a Fisher distribution with ν_1 and ν_2 degrees of freedom, respectively.

A distinction is made between SIMO-data estimates and MIMO-data estimates above. The reason is that there is separate information concerning the noise filter W from each SIMO experiment. For example, the noise coloring filter $W(z^{-1}, n_2)$ can be estimated by monitoring the noise in sensor n_2 in each of q separate SIMO experiments. Hence, one can generate q separate estimates of $W(z^{-1}, n_2)$ (denoted above as $\widetilde{W}(z^{-1}, n_2|n_1)$; $n_1 = 1, ..., q$), or alternatively, combine all data sets into a single MIMO-data estimate (denoted as $\widehat{W}(z^{-1}, n_2)$).

Clearly, MIMO-data estimates (25) of the noise filter will be more accurate than the SIMO-data estimates (22) since they use more data (i.e., note the double summation in the numerator of (25)). However, the MIMO-data plant error statistics (27) will be statistically *dependent* across each row of the transfer function, since they involve a common noise filter estimate $\widehat{W}(z^{-1}, n_2)$. In contrast, plant error statistics based on SIMO-data (24) will remain *independent* across each row of the transfer function, since SIMO data is statistically independent from one experiment to the next. For combining probabilistic events, statistical independence is often preferred.

6. MULTIVARIABLE RESULTS

A statistical multivariable plant set estimate can be directly obtained from Lemma 2 if one can find ϵ_i such that the event,

$$E_i: \quad \overline{\sigma}(\mathcal{P}(\omega_i) - \widehat{P}(\omega_i)) \leq \epsilon_i \tag{28}$$

is satisfied with at least probability $1 - \kappa$ at each grid point i. Such values of ϵ_i will be determined in this section under Assumptions 1-5.

Consider the well known Frobenious norm bound on the square of the maximum singular value,

$$\overline{\sigma}(\mathcal{P}(\omega_i) - \widehat{P}(\omega_i))^2 \leq ||\mathcal{P}(\omega_i) - \widehat{P}(\omega_i)||_f^2 \tag{29}$$

$$= \sum_{n_2=1}^{r} \sum_{n_1=1}^{q} |\mathcal{P}(\omega_i, n_2, n_1) - \widehat{P}(\omega_i, n_2, n_1)|^2 \tag{30}$$

where $||X||_f \stackrel{\Delta}{=} tr\{X^T X\}^{1/2}$. Using the SIMO-data error statistics (24) each of the terms in (30) can be overbounded to probability $1 - \gamma$ by,

$$|\mathcal{P}(\omega_i, n_2, n_1) - \widehat{P}(\omega_i, n_2, n_1)|^2 \leq \rho_i^{1-\gamma}(n_2, n_1) \tag{31}$$

where,

$$\rho_i^{1-\gamma}(n_2, n_1) = \frac{2|\widehat{W}(\omega_i, n_2|n_1)|^2}{\alpha_i(n_1)m(n_1)N_s} \cdot F_{1-\gamma}(2, 2m(n_1) - 2) \tag{32}$$

and $F_{1-\gamma}(\nu_1, \nu_2)$ denotes the %$(1 - \gamma)$ x 100 percentile for the Fisher distribution.

The events (31) are statistically independent for $n_1 \neq n_2$ since,

1) The errors in each row of $\widehat{P}$ are independent by the diagonal structure of noise W

2) The errors in each column of $\widehat{P}$ are independent by the SIMO data acquisition procedure and the use of SIMO-data estimates (24) in the calculation of the percentiles in (32).

Hence, from *(30)* and (31,) the event,

$$E_i: \quad \overline{\sigma}(\mathcal{P}(\omega_i) - \widehat{P}(\omega_i))^2 \leq \sum_{n_2=1}^{r} \sum_{n_1=1}^{q} \rho_i^{1-\gamma}(n_2, n_1) \overset{\triangle}{=} \epsilon_i \tag{33}$$

holds with probability,

$$(1 - K) = (1 - \gamma)^{q \cdot r} \tag{34}$$

as desired. Furthermore, under Assumption 5, the events $E_i\, i = 1,..., N$ in (33) are st artistically independent from one frequency to the next. Hence, one can use the results of Lemma "2 to generate an additive uncertainty overbound $\ell_A^{1-\alpha}(\omega)$ to confidence $1 - \alpha$ using the formula $1 - \alpha = (1 - \kappa)^{n_s}$.

Using the LPSOF algorithm [19] to overbound the *nonparametric* additive error $\ell_A^{1-\alpha}(\omega)$ gives an additive uncertainty weighting filter $W_A(z^{-1}) = w_A(z^{-1})$. I in the form of a parametric filter w_A times the identity matrix. The additive error is now in the desired parametric form for use with control design software.

7. CONCLUSIONS

The present paper extends statistical plant estimation approaches to the multivariable case. This approach produces a nominal plant estimate P^o and the additive uncertainty weighting filter W_A . I, such that the true plant lies in the additive uncertainty set $\mathcal{P} = P^o + W_A \Delta$ to a prescribed statistical confidence %$(1 - \alpha)$ x 100. Hence, any controller designed to be robust with respect to P^o and W_A will work on the true system to the same %$(1 - \alpha)$ x 100 statistical confidence.

The main device used here to effect the extension to the multivariable case is to overbound the maximum singular value of a matrix X by its Frobenious norm $\|X\|_f = Tr\{X^T X\}^{1/2}$, i.e.,

$$\frac{1}{\sqrt{\min\{q, r\}}} \|X\|_f \leq \overline{\sigma}(X) \leq \|X\|_f$$

It is seen that this bound can become conservative if both the number of inputs q and outputs r of the plant increases. However, the maximum singular value of a random matrix has very complicated statistics, while the Frobenious norm is characterized completely in terms of second-order statistics. This property has enabled the development of statistical uncertainty bounds based directly on the measured data.

The reader is warned that the use of M, ρ information to overbound the *interpolation error* in Lemma 2, can be wildly conservative in practice. This is especially true when applied to lightly damped systems. A reasonable practical approach is to choose a "good" frequency grid based engineering judgement, and to neglect the interpolation error in the computation of the uncertainty.

The restriction of having to know the noise filters $W(z^{-1})$ has been relaxed compared to earlier treatments. This comes at a cost of having to choose a sufficiently large input period $N_s T$ compared to the time constants of W and its inverse. A precise characterization of the error when this condition (i.e., Assumption 5) is violated is of interest but is not presently available. The requirement (Assumption 3) that W is diagonal is also restrictive, and effort is underway to relax this condition.

The present method is based on a multisinusoidal input signal, which is somewhat restrictive compared to other approaches which allow arbitrary inputs. However, there is a great deal of flexibility in designing the spectrum of the multisinusoidal excitation, which can be useful in applications. Furthermore, it is expected that the multisinusoidal signal can be applied at low levels over long periods of time, in order to extract plant knowledge with minimal impact to normal system operations.

ACKNOWLEDGEMENT'S

REFERENCES

[1] G .J. Balas, J.C. Doyle, K. Glover, A.K. Packard, R. Smith, *H-Infinity and Mu Control Analysis : Mu- Tools Manual.* The MathWorks Inc. 1990.

[2] D.S. Bayard, "Multivariable frequency domain identification via 2-Norm minimization," Proc. American Control Conference, Chicago, Illinois, pp. 1253-1257, June 1992.

[3] D.S. Bayard, " Statistical plant set estimation using Schroeder-phased multisinusoidal input designs," J. Applied Mathematics and Computation, vol. 58, pp. 169-198, 1993.

[4] D. S, Bayard, "An algorithm for state-space frequency domain identification without windowing distortions," Proc. 31st IEEE Conference on Decision and Control, Tucson, Arizona, December 1992.

[5] D.S Bayard, "Multivariable plant and disturbance identification using multisinusoidal input excitation," JPL Internal Document, in preparation.

[6] D.S. Bayard, Y. Yam, E. Mettler, "A criterion for joint optimization of identification and robust control," IEEE Transactions on Automatic Control, *Special Mini-Issue on System Identification for Control Design,* vol. 37, no. 7, pp. 986-991, July 1992.

[7] R..Y. Chiang and M.G. Safonov, *Robust- Control Toolbox.* The MathWorks, Inc. 1988.

[8] D.K. de Vries, and P.M.J. Van den Hof, "Quantification of model uncertainty from data: input design, interpolation, and connection with robust control design specification," Proc. American Control Conference, Chicago, Illinois, pp. 3170-3175, June 1992.

[9] D.K. de Vries, and P.M.J. Van den Hof, "Quantification of model uncertainty from experimental data: a mixed deterministic-probabilistic approach," Internal Report, Dept. Mech. Eng., Delft University Techn., 1992.

[10] G. Gu and P.P. Khargonekar, "A class of algorithms for identification in H_∞," Automatic, vol. 28, pp. 299-312, 1992.

[11] P. Guillaume, J. Schoukens, R. Pintelon, and I. Kollar, "Crest-factor minimization using nonlinear Chebyshev approximation methods,)' IEEE Trans. Instr. and Meas., vol. 40, no. 6, December 1991.

[12] G.C. Goodwin and M.E. Salgado, "Quantification of uncertainty in estimation using an embedding principle)" Proc. American Control conference, pp. 1416-1421, 1989.

[13] A.J. Helmicki, C.A. Jacobson and C.N. Nett, "Identification in H_∞: a robustly convergent, nonlinear algorithm," Proc. American Control Conference, pp. 386-391, 1990.

[14] R.L. Kosut, "Identification of systems with parametric and nonparametric uncertainty," American Control Conference, San Diego, CA June 1990.

[15] V. Kucera, *Discrete Linear Control: The Polynomial Equation Approach*, John Wiley and Sons, Chichester, 1979.

[16] R.O. Lamaire, L. Valavani, M. Athans, and G. Stein, "A frequency-domain estimator for use in adaptive control systems, " Automatic, vol. 27, no. 1, pp. 23-38, 1991.

[17] P.J. Parker and R.R. Bitmead, "Adaptive frequency response identification," Proc. 26th Conf. on Decision and control, pp. 348-353, Los Angeles, CA 1987.

[18] C.K. Sanathanan and J. Koerner, "Transfer function synthesis as a ratio of two complex polynomials," *IEEE Trans. Auto. Contr.*, vol. 8, pp.56-58, 1963.

[19] R.E. Scheid, D.S. Bayard, Y. Yam, "A Linear Programming Approach to Characterizing Norm Bounded Uncertainty From Experimental Data," American Control Conference, Boston, MA, pp. 1956-1958, June 1991.

[20] M.R. Schroeder, "Synthesis of low peak-factor signals and binary sequences of low auto-correlation," IEEE Trans. Information Theory, Jan. 1970.

[21] R.C. Younce and C.E. Rohrs, "Identification with non-parametric uncertainty," IEEE Trans. Auto. Contr., vol. 37, pp. 715-728, 1992,

[22] P, Young, and Patton, R. J., "Comparison of test signals for aircraft frequency domain identification," AIAA J. Guidance, Dynamics, and Control, May-June 1990.